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Failure-Finding Frequency for a Repairable System Subject to Hidden Failures

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This paper addresses the problem of selecting a suitable failure-finding maintenance policy for repairable systems. We will consider hidden failures that do not interrupt aircraft operation when they occur, such as failures of warning devices or backup components. We will study both corrective maintenance actions, carried out after item failure, and periodic failure finding, designed to check whether a system still works. Based on our probabilistic analytic developments, the optimal maintenance policy is then obtained as a solution of an optimization problem, in which the maintenance cost rate is the objective function and the risk of corrective maintenance is the constraint function. Finally, we will present an application of our methodology to a real-world case provided by Airbus Industries.

Nomenclature

C_{check} = cost to perform the check during failure-finding maintenance
C_{CM} = cost of corrective maintenance
C_{FF} = cost of failure-finding maintenance to restore the hidden system following detection during check
FS = fleet size, given input
I^{N+1} = residual length of time between the last failure-finding tasks at t^{N+1} and $t^{N+2} = T$ for optimization studies, $I^{N+1} = [t^{N+1}; T]$
k = index of failure-finding interval from 0 to N within the calculation horizon T
$NCM^{j}_{\mathrm{aircraft}}(t)$ = counting process of corrective maintenance actions due to unscheduled failures for one aircraft in the interval I^j from t^j to t, $\forall\ t \in I^j$, $j \in [0; N+1]$
$NCM^{j}_{\mathrm{fleet}}(t)$ = mean number of corrective maintenance actions due to unscheduled failures on the fleet from t^j to t, $\forall\ t \in I^j$, $j \in [0; N+1]$
NFF^{j}_{fleet} = mean number of detected failures on the fleet during failure-finding tasks performed at time t^{j+1}, $j \in [0; N+1]$
$P^{j}_{\mathrm{no\ CM}}$ = probability of avoiding corrective maintenance actions during interval I^j, $j \in [0; N+1]$
T = time horizon of the fleet maintenance policy; T is a given input
$Y^j(t)$ = state variable of the system with respect to its hidden or undetected failures at time t in the interval I^j, $j \in [0; N+1]$
$\lambda_{\mathrm{demand}}(t)$ = occurrence rate of the operational demand at time t; $\forall\ t \in T$, $\lambda_{\mathrm{demand}}(t)$ is a given input
$\lambda^{j}_{\mathrm{CM}}(t)$ = occurrence rate of corrective maintenance actions at time t in the interval I^j, $j \in [0; N+1]$
$\lambda^{j}_{\mathrm{system}}(t)$ = failure rate of the system whose failure is hidden on the interval I^j, $j \in [0; N+1]$; $\lambda^{0}_{\mathrm{system}}(t) \forall\ t \in T$ is the given failure rate of this system without any corrective or failure-finding maintenance

I. Introduction

MUCH of what has been written to date on the subject of maintenance strategies [1,2] refers to predictive, preventive, and corrective maintenance. Far less attention has been paid to the case of failure-finding maintenance for systems with hidden failures.

A hidden failure [3] is a failure not evident to the crew or operator during the performance of normal duties. These failures occur in such a way that the operator does not know that the item is in a failed state unless some other operational demand (additional failure, trigger event) also occurs. For instance, if a standby radio fails, no one would be aware of the fact because under normal circumstances the active radio would still be working. In other words, the failure of the

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standby radio on its own has no direct impact until the active radio also fails. Generally, hidden failure affects backup and protection systems, such as safety valves (e.g., a shutdown valve or relief valve) or sensors (e.g., fire/gas detector, pressure or level sensor), which are designed to be activated upon operational demand to protect people, the environment, or to keep a given function. These systems are common in industrial safety and protection systems; examples are presented in [4] with standby devices.

Failure-finding maintenance is far from being negligible. According to Moubray [5], if reliability centered maintenance is correctly applied to almost any modern, complex industrial system, it is not unusual to find that up to 40% of failure modes fall into the hidden category. Furthermore, up to 80% of these failure modes require failure finding. As a result, up to one third of the tasks generated by comprehensive, correctly applied maintenance strategy development programs are failure-finding tasks. In this case, we have to deal in particular with both corrective maintenance actions and periodic failure-finding tasks.

Corrective maintenance tasks are carried out after an item has failed. The purpose of corrective maintenance is to restore the item to a functioning state as soon as possible, either by repairing or replacing the failed item, or by switching to a redundant item. Corrective maintenance is also called breakdown maintenance or run-to-failure maintenance. To sum up, corrective maintenance refers to the actions performed, as a result of failure, to restore an item to a specified condition [3].

In contrast, failure-finding tasks are carried out to reveal hidden failures that have already occurred. For example, consider a smoke detector as an emergency system where the audible alarm sounds only when smoke is present. Failure of the smoke detector during normal operation would constitute a hidden failure. The failure of the smoke detector would only become evident when smoke was present and it failed to sound. A failure-finding task for the smoke detector would be to periodically check the fire detection circuit to see if it is operational (blowing smoke at the detector and checking if the warning sounds). Another example is a pressure switch designed to shut down a machine when the lubricating oil pressure drops below a certain level. Switches of this type should be checked regularly by dropping the oil pressure to the appropriate level and checking whether the machine shuts down.

Ideally, we have to find a failure-finding task interval that ensures 100% availability of the protection or backup system when operational demand occurs. In this ideal case, there are no corrective maintenance tasks and the number of failure-finding tasks is infinite. In practice, it is impossible to achieve a 100% availability of the hidden system, because of both technical feasibility and the costs induced by the high number of failure-finding tasks. Therefore, in real-world cases of failure without safety impact, the problem is to determine an optimal frequency of failure-finding tasks that achieves the best tradeoff between the cost of corrective tasks and the cost of failure-finding tasks. For some systems, due to safety or operational reasons, constraints on system availability have also to be taken into account in the optimization process and redesign may be necessary.

The purpose of this paper is to develop a precise framework to determine the optimal failure-finding maintenance frequency for repairable systems put in operation at time $t = 0$ for a finite time horizon. It is organized as follows. In Sec. II, based on in-service aircraft operation, we present our basic working assumptions. Section III is devoted to mathematical developments. We give equations for computing the mean number of both corrective and failure-finding maintenance actions over a finite time horizon. Based on these results, we present the optimization problem that defines the optimal frequency of failure-finding tasks. Section IV deals with constant failure rates. Simple analytical formulas are then derived from those of Sec. III. They provide a clear understanding of the influence of each reliability parameter on the optimization process result. In Sec. V, a numerical example provided by Airbus Industries is given on a pressure relief valve to illustrate how the model can be used on a real-world case. Finally, in Sec. VI, we conclude and open further perspectives. Figure 1 shows how the index is used throughout the article with regard to time reference.

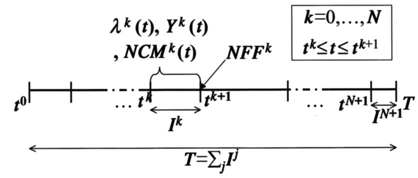

Fig. 1 Index with regard to time reference.

II. Assumptions

The following modeling assumptions are based on common airline practice.

1) The operational demand is a general, nonhomogenous Poisson process [6]. In particular, the rate of occurrence of the demand can be a function of time and there will be no more than one demand at any given time.

2) The failure rate of a system with a hidden failure is a function of time (e.g., infant mortality, aging effect, etc.). Failures of all fleet systems are assumed to be statistically independent with the same probability distribution.

3) If the hidden system checked during a failure-finding task is in an operating state, nothing is done.

4) If the hidden system checked during a failure-finding task is in a failed state, it is replaced by a new component of the same type or restored to a "good as new" condition. This means that the failure time distribution of the repaired or the new system is identical to that of the previous one at $t = 0$.

5) In case of corrective maintenance, the breakdowns are minimally repaired. With minimal repair, a failed item is returned to operation with the same effective age it had immediately before failure.

6) There is no time value for money. Money available now is not worth more than the same amount in the future regardless of its potential earning capacity.

III. Analytic Development

In this section we define the model of failure-finding maintenance as a Markovian process. From this model, we derive the formula of the system failure rate. Then, we estimate the mean number of maintenance actions over a fleet, either corrective or during failure-finding tasks. We also assess the probability of avoiding corrective maintenance actions over an interval. Finally, we state the optimization problem that allows determination of the optimal failure-finding frequency, minimizing maintenance costs and satisfying an acceptable level of corrective maintenance frequency.

A. Definition of System State and Failure Rate

The type of system studied here can be modeled within an interval by a simple Markovian graph, as shown in Fig. 2.

$\forall\ t \in I^j$, $j \in [0; N+1]$, the state variable $Y^j(t)$ of a system with respect to its hidden failures, is

1) $Y^j(t) = 1$, if the system is functioning at time t;

2) $Y^j(t) = 0$, if the system item is not functioning at time t.

$\forall\ t \in I^j$, with the assumption of minimal repair during corrective maintenance, we have the following differential equation:

$$\dot{\overline{\begin{pmatrix} P[Y^j(t)=1] \\ P[Y^j(t)=0] \end{pmatrix}}} = \begin{pmatrix} -\lambda^j_{\text{system}}(t) & \lambda_{\text{demand}}(t) \\ \lambda^j_{\text{system}}(t) & -\lambda_{\text{demand}}(t) \end{pmatrix} \begin{pmatrix} P[Y^j(t)=1] \\ P[Y^j(t)=0] \end{pmatrix} \tag{1}$$

With

$$P[Y^j(t)=1] + P[Y^j(t)=0] = 1 \tag{2}$$

and the initial conditions

$$\begin{pmatrix} P[Y^j(t^j)=1] \\ P[Y^j(t^j)=0] \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{3}$$

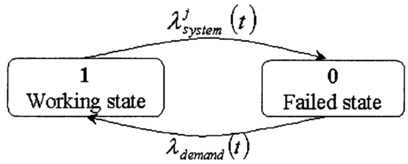

Fig. 2 Markovian transition diagram.

Let us define

$$A^j(t) = \begin{pmatrix} -\lambda^j_{\text{system}}(t) & \lambda_{\text{demand}}(t) \\ \lambda^j_{\text{system}}(t) & -\lambda_{\text{demand}}(t) \end{pmatrix}$$

Then the analytical solution of the previous Eqs. (1–3) is given by

$$\forall\ t \in I^j, \qquad \begin{pmatrix} P[Y^j(t)=1] \\ P[Y^j(t)=0] \end{pmatrix} = \exp\left(\int_{t_j}^{t} A^j(\tau)\,\mathrm{d}\tau\right)\begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{4}$$

To solve it, we can use discretization schemes, such as Euler or Runge–Kutta [7].

Because of assumptions made on corrective and failure-finding maintenance, it is now straightforward to formulate the system failure rate as

$$\begin{aligned} &1 \le j \le N, \quad \forall\ t \in I^j, \\ &\lambda^j_{\text{system}}(t) = \lambda^{j-1}_{\text{system}}(t) \times P[Y^{j-1}(t^j)=1] + \lambda^0_{\text{system}}(t-t^j) \\ &\quad \times P[Y^{j-1}(t^j)=0] \end{aligned} \tag{5}$$

where $\lambda^0_{\text{system}}(t)$ $\forall\ t \in T$ is the given failure rate of this system without any corrective or failure-finding maintenance and $\lambda_{\text{demand}}(t)$ $\forall\ t \in T$ is the given occurrence rate of the operational demand.

B. Mean Numbers of Maintenance Actions

Basically the system studied in this paper can fail to operate 1) in the presence of an operational demand generating corrective maintenance and 2) in the absence of operational demands generating failure-finding maintenance.

The probability to detect a failure for one aircraft during failure-finding tasks performed at time t^{j+1} $j \in [0; N+1]$ is given by

$$NFF^j_{\text{aircraft}} = P[Y^j(t^{j+1}) = 0] \tag{6}$$

Besides $NCM^j_{\text{aircraft}}(t)$, the mean number of corrective maintenance actions on $[t^j; t]$ per aircraft in the interval I^j is a renewal process. Its rate of occurrence can be easily computed from $\lambda^j_{\text{CM}}(t) = \lambda_{\text{demand}}(t) \times P[Y^j(t)=0]$ $\forall\ t \in I^j$.

Then we have

$$\forall\ t \in I^j, \qquad NCM^j_{\text{aircraft}}(t) = \int_{t^j}^{t} \lambda^j_{\text{CM}}(\tau)\,\mathrm{d}\tau \tag{7}$$

These results are directly expanded to a fleet through the following formulas $\forall\ j \in [0; N+1]$:

$$NFF^j_{\text{fleet}} = FS \times NFF^j_{\text{aircraft}} \tag{8}$$

$$\forall\ t \in I^j, \qquad NCM^j_{\text{fleet}}(t) = FS \times NCM^j_{\text{aircraft}}(t) \tag{9}$$

C. Probability of Avoiding Corrective Maintenance Actions Between Two Failure-Finding Tasks

The event no corrective maintenance action during interval I^j $\forall\ j \in [0; N+1]$ is divided in two distinct events: 1) no system failure within interval I^j and 2) system failure within I^j, but no operational demand within I^j after system failure.

Therefore we can write

$$P^j_{\text{no CM}} = P(\text{no system failure within } I^j) + P\begin{pmatrix} \text{system failure within } I^j \\ \cup \\ \text{no process demand within } I^j \text{ after system failure} \end{pmatrix} \tag{10}$$

We are going to detail each of these two probabilities.

To begin with, the probability of no system failure within I^j is given by

$$P(\text{no system failure within } I^j) = \exp\left(-\int_{t^j}^{t^{j+1}} \lambda^j_{\text{system}}(\tau)\,\mathrm{d}\tau\right) \tag{11}$$

A first order approximation of the probability of first system failure between t and $t + \mathrm{d}t$ at time t is given (here) by the density function multiplied by $\mathrm{d}t$. It is well known that the density function of a first system failure is equal to the failure rate multiplied by the survival function:

$$\forall\ t \in I^j, \qquad \lambda^j_{\text{system}}(t) \times \exp\left(-\int_{t^j}^{t} \lambda^j_{\text{system}}(\tau)\,\mathrm{d}\tau\right) \tag{12}$$

Now we assess the probability of no system demand after system failure at time t.

The number of operational demands $\{N_{\text{demand}}(t),\ t \ge 0\}$ is assumed to be a nonhomogeneous Poisson process (NHPP) with rate function $\lambda_{\text{demand}}(t)$ for $t \ge 0$. From the properties of NHPP [6], we have the following equality:

$$\begin{aligned} &\forall\ t, \tau \in I^j \quad \text{with} \quad \tau \le t, \\ &P[N_{\text{demand}}(t) - N_{\text{demand}}(\tau) = 0] = \exp\left(-\int_{\tau}^{t} \lambda_{\text{demand}}(u)\,\mathrm{d}u\right) \end{aligned} \tag{13}$$

To sum up, the probability of no operational demand after system failure is

$$\begin{aligned} &P\begin{pmatrix} \text{system failure within } I^j \\ \cup \\ \text{no process demand within } I^j \text{ after system failure} \end{pmatrix} \\ &= \int_{t^j}^{t^{j+1}} \lambda^j_{\text{system}}(t) \times \exp\left(-\int_{t^j}^{t} \lambda^j_{\text{system}}(\tau)\,\mathrm{d}\tau\right) \\ &\quad \times \exp\left(-\int_{t}^{t^{j+1}} \lambda_{\text{demand}}(\tau)\,\mathrm{d}\tau\right)\mathrm{d}t \end{aligned} \tag{14}$$

As a result the probability of avoiding corrective maintenance actions over an interval I^j is the sum of Eqs. (11) and (14):

$$\begin{aligned} P^j_{\text{no CM}} &= \exp\left(-\int_{t^j}^{t^{j+1}} \lambda^j_{\text{system}}(\tau)\,\mathrm{d}\tau\right) + \int_{t^j}^{t^{j+1}} \lambda^j_{\text{system}}(t) \\ &\quad \times \exp\left(-\int_{t^j}^{t} \lambda^j_{\text{system}}(\tau)\,\mathrm{d}\tau\right) \times \exp\left(-\int_{t}^{t^{j+1}} \lambda_{\text{demand}}(\tau)\,\mathrm{d}\tau\right)\mathrm{d}t \end{aligned} \tag{15}$$

D. Optimization of Failure-Finding Frequency

The problem here is to determine the optimal interval length I^k between failure-finding tasks over a finite time horizon $[0, T]$ (e.g., the time an airline holds an aircraft). Under a risk constraint on the probability of avoiding corrective maintenance, the optimal objective is to minimize the maintenance cost of the system by balancing failure-finding tasks and corrective actions. A model to optimize maintenance policies by minimizing the system cost rate with availability constraint is presented in [8].

Compared to corrective maintenance, failure-finding maintenance is scheduled and therefore cheaper, whereas failure during operation might be costly and dangerous. However, a large number of failure-finding checks can also lead to prohibitive costs. Therefore, the problem is to determine the best optimal length I^k that ensures the best tradeoff.

For the sake of simplicity and because in real-world cases of aircraft maintenance the intervals are constant, we assume in this section that the intervals between two successive scheduled tasks are constant (i.e., $I^k = I$ and $t^k = k \times I$). Note that the maintenance optimization problem can be extended in a straightforward way to variable interval length. In this case, we have to deal with a multivariable optimization problem.

The objective function of this optimization problem is the expected maintenance cost rate on the horizon period T. Its numerator is the sum of the maintenance costs on each interval $I^k = I$ plus the residual length interval I^{N+1}. For each interval, maintenance costs take into account 1) the cost of corrective maintenance to restore an item following functional failure; 2) the cost of failure-finding maintenance to restore an item following detection during check; and 3) the cost to perform the check during a failure-finding task to identify any potential failure.

The denominator term is the T horizon interval length. There are two constraints to this optimization problem. This interval length must be greater than or equal to zero and the probability of having no corrective maintenance must be greater than a given threshold δ. This threshold corresponds to the maximum probability of an unscheduled event that the airline tolerates. Note that $\delta = 0$ means that the only criterion is economic.

The optimization problem can be expressed as

$$\min C(I) \tag{16}$$

Subject to

$$I \geq 0 \tag{17}$$

$$\min_{0\leq j\leq N+1} P^j_{\text{no CM}} \geq \delta \tag{18}$$

With

$$C(I) = \begin{cases} \frac{\sum_{j=0}^{N}(C_{\text{CM}}\times NCM^j_{\text{fleet}}(j\times I+I)+C_{\text{FF}}\times NFF^j_{\text{fleet}}+C_{\text{check}})}{T}, & \text{if } I^{N+1}=0 \\ \frac{\sum_{j=0}^{N}(C_{\text{CM}}\times NCM^j_{\text{fleet}}(j\times I+I)+C_{\text{FF}}\times NFF^j_{\text{fleet}}+C_{\text{check}})}{T-I^{N+1}} & \\ +\frac{C_{\text{CM}}\times NCM^{N+1}_{\text{fleet}}+C_{\text{FF}}\times NFF^{N+1}_{\text{fleet}}+C_{\text{check}}}{I^{N+1}}, & \text{if } I^{N+1}>0 \end{cases} \tag{19}$$

This optimization problem generally has no analytical solution. The I^* optimal solution can be obtained by using adequate nonlinear programming methods [9].

IV. Application for Constant Failure Rate

In this section, we developed formulas for a system in which hidden failures can occur randomly, that is, with a constant failure rate [10,11]. This assumption is widely used in the real aeronautical world when we have to deal with systems during their useful life period. With constant failure for both system and operational demand, simple analytical formulas are derived from those of Sec. III. They provide a clear understanding of the influence of each input parameter on the optimization process.

A. Mean Numbers of Maintenance Actions

The system with Eqs. (1–3) can be resolved analytically. The probability that the item is functioning at time t is

$$\forall\ t \in I^j,$$

$$P[Y^j(t)=1] = \frac{\lambda_{\text{demand}}}{\lambda_{\text{demand}}+\lambda^0_{\text{system}}} + \frac{\lambda^0_{\text{system}}}{\lambda_{\text{demand}}+\lambda^0_{\text{system}}} \times \exp[-(\lambda_{\text{demand}}+\lambda^0_{\text{system}})\times(t-t^j)] \tag{20}$$

The probability that the item is not functioning at time t is

$$\forall\ t \in I^j,$$

$$P[Y^j(t)=0] = \frac{\lambda^0_{\text{system}}}{\lambda_{\text{demand}}+\lambda^0_{\text{system}}} - \frac{\lambda^0_{\text{system}}}{\lambda_{\text{demand}}+\lambda^0_{\text{system}}} \times \exp[-(\lambda_{\text{demand}}+\lambda^0_{\text{system}})\times(t-t^j)] \tag{21}$$

We can now write the Eqs. (8) and (9) analytically. The mean numbers of detected failures on the fleet during failure-finding tasks performed at time t^{k+1} and t are given, respectively, by

$$\forall\ k \in [0;N]$$

$$NFF^k_{\text{fleet}} = \frac{FS\times\lambda^0_{\text{system}}}{\lambda_{\text{demand}}+\lambda^0_{\text{system}}}\{1-\exp[-(\lambda_{\text{demand}}+\lambda^0_{\text{system}})\times I^k]\} \tag{22}$$

$$NFF^{N+1}_{\text{fleet}} = \begin{cases} \frac{FS\times\lambda^0_{\text{system}}}{\lambda_{\text{demand}}+\lambda^0_{\text{system}}}\{1-\exp[-(\lambda_{\text{demand}}+\lambda^0_{\text{system}})\times I^{N+1}]\} & \text{if } I^{N+1}>0 \\ 0 & \text{if } I^{N+1}=0 \end{cases} \tag{23}$$

with

$$I^{N+1} = T - \sum_{k=0}^{N} I^k$$

The mean number of corrective maintenance actions due to unscheduled failures on the fleet from t^j to t, $\forall\ t \in I^j$ is given by

$$NCM^j_{\text{fleet}}(t) = \frac{FS\times\lambda_{\text{demand}}\times\lambda^0_{\text{system}}}{\lambda_{\text{demand}}+\lambda^0_{\text{system}}}\int_{t^j}^{t}\{1-\exp[-(\lambda_{\text{demand}}+\lambda^0_{\text{system}})\times(\tau-t^j)]\}\,\mathrm{d}\tau \tag{24}$$

It is straightforward to deduce the total number of corrective maintenance actions over interval I^k and I^{N+1}.

$$NCM^k_{\text{fleet}}(t^{k+1}) = \frac{FS\times\lambda_{\text{demand}}\times\lambda^0_{\text{system}}}{\lambda_{\text{demand}}+\lambda^0_{\text{system}}} \times\left[I^k + \frac{\exp[-(\lambda_{\text{demand}}+\lambda^0_{\text{system}})\times I^k]-1}{\lambda_{\text{demand}}+\lambda^0_{\text{system}}}\right] \tag{25}$$

$$NCM^{N+1}_{\text{fleet}} = \begin{cases} \frac{FS\times\lambda_{\text{demand}}\times\lambda^0_{\text{system}}}{\lambda_{\text{demand}}+\lambda^0_{\text{system}}}\left[I^{N+1}+\frac{\exp[-(\lambda_{\text{demand}}+\lambda^0_{\text{system}})\times I^{N+1}]-1}{\lambda_{\text{demand}}+\lambda^0_{\text{system}}}\right] & \text{if } I^{N+1}>0 \\ 0 & \text{if } I^{N+1}=0 \end{cases} \tag{26}$$

Note the special cases where there are no failure-finding tasks. In this case, the equipment is deliberately run to failure and the previous probabilities become

$$\lim_{t\to+\infty} P[Y(t)=1] = \frac{\lambda_{\text{demand}}}{\lambda_{\text{demand}}+\lambda^0_{\text{system}}} \tag{27}$$

$$\lim_{t\to+\infty} P[Y(t)=0] = \frac{\lambda^0_{\text{system}}}{\lambda_{\text{demand}}+\lambda^0_{\text{system}}} \tag{28}$$

$$\lim_{t^{j+1}\to+\infty} NFF^j_{\text{fleet}} = \frac{FS\times\lambda^0_{\text{system}}}{\lambda_{\text{demand}}+\lambda^0_{\text{system}}} \quad \text{with} \quad t^j=0 \tag{29}$$

$$\lim_{t^{j+1}\to+\infty} NCM^{j}_{\text{fleet}}(t^{j+1}) = +\infty \quad \text{with} \quad t^{j}=0 \tag{30}$$

B. Probability of Avoiding Corrective Maintenance Actions Between Failure-Finding Tasks

With the assumption of a constant failure rate, Eq. (15) can be rewritten as follows:

$$P^{j}_{\text{no CM}} = \exp\left(-\int_0^{I^j} \lambda^0_{\text{system}}\, d\tau\right) + \int_{t^j}^{t^{j+1}} \lambda^0_{\text{system}} \times \exp\left(-\int_0^{t-t^j} \lambda^0_{\text{system}}\, d\tau\right) \times \exp\left(-\int_{t-t^j}^{I^j} \lambda_{\text{demand}}\, d\tau\right) dt \tag{31}$$

Equation (31) can be developed in a straightforward way and we obtain the following probability of facing no risk of corrective maintenance over I^k:

$$P^{k}_{\text{no CM}} = \exp(-I^k \times \lambda^0_{\text{system}}) + \frac{\lambda^0_{\text{system}}}{\lambda_{\text{demand}} - \lambda^0_{\text{system}}} \times [\exp(-I^k \times \lambda^0_{\text{system}}) - \exp(-I^k \times \lambda_{\text{demand}})] \tag{32}$$

$$P^{N+1}_{\text{no CM}} = \begin{cases} \exp(-I^{N+1} \times \lambda^0_{\text{system}}) + \frac{\lambda^0_{\text{system}}}{\lambda_{\text{demand}}-\lambda^0_{\text{system}}} \times [\exp(-I^{N+1} \times \lambda^0_{\text{system}}) - \exp(-I^{N+1} \times \lambda_{\text{demand}})] & \text{if } I^{N+1} > 0 \\ 0 & \text{if } I^{N+1} = 0 \end{cases} \tag{33}$$

When there is no failure-finding task, that is, $t^j = 0$ and $t^{j+1} \to +\infty$, the limit of $P^{j}_{\text{no CM}}$ is

$$\lim_{t^{j+1}\to+\infty} P^{j}_{\text{no CM}} = 0 \quad \text{with} \quad t^{j}=0 \tag{34}$$

C. Optimization of Failure-Finding Frequency

Based on the formulas of the previous sections, we are now able to compute the analytic sensitivities of the objective and constraint functions of the optimization problem (16–18).

Note that when there is no failure-finding task, that is, $I \to +\infty$, the limit of $C(I)$ is

$$\lim_{I\to+\infty} C(I) = C_{\text{CM}} \times \frac{FS \times \lambda_{\text{demand}} \times \lambda^0_{\text{system}}}{\lambda_{\text{demand}} + \lambda^0_{\text{system}}} \tag{35}$$

V. Numerical Example

A. Case Study Definition

To illustrate the formulas developed, we considered a pressure relief valve. The component is a typical failure-finding scenario because the valve serves as a protection device. Its conditional failure property is unrelated to age and it does not exhibit infant mortality (it is a mature component).

Pressure relief valves are designed to provide protection from overpressure in steam, gas, air, and liquid pipes. As a result, the failure of the valve remains hidden if there is no overpressure. Moreover there is a process demand of the valve every time safe pressures are exceeded. In this case, the valve releases steam to drop the pressure to a preset level. The corrective maintenance actions will be performed every time the system exceeds a preset level, that is, every time the valve fails to ensure protection.

We consider the following numerical data:

1) The mean time between value failure = 5000 flight hours (FH), hence $\lambda^0_{\text{system}} = 1/5000$.

2) The valve is used once every 900 flight hours, hence $\lambda_{\text{demand}} = 1/900$.

3) There are 10 aircraft in the fleet, hence $FS = 10$.

4) The cost to perform the check during failure-finding maintenance is $C_{\text{Check}} = 15$.

5) The cost of failure-finding maintenance is $C_{\text{FF}} = 60$.

6) The cost of corrective maintenance is $C_{\text{CM}} = 150$.

7) The computation period is $T = 10,000$ flight hours.

B. Impact of Failure-Finding Frequency

In this section we illustrate the formulas presented in Sec. IV. The failure-finding interval length varies from 100 flight hours to T.

The first shape, as shown in Fig. 3, is the maintenance cost rate $C(I)$ as a function of I. It shows how the length of the failure-finding interval impacts the maintenance cost. This is a U curve.

The curve in Fig. 4 presents as a function of interval length the probability of avoiding corrective maintenance actions during a failure-finding interval. This curve shows that, when the interval length increases, the probability of having corrective maintenance

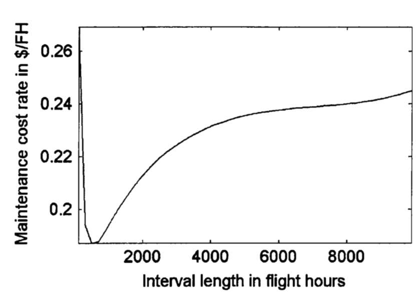

Fig. 3 Maintenance cost as a function of failure-finding interval length.

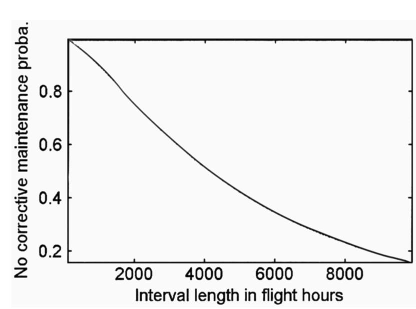

Fig. 4 Probability of no corrective maintenance actions as a function of failure-finding interval length.

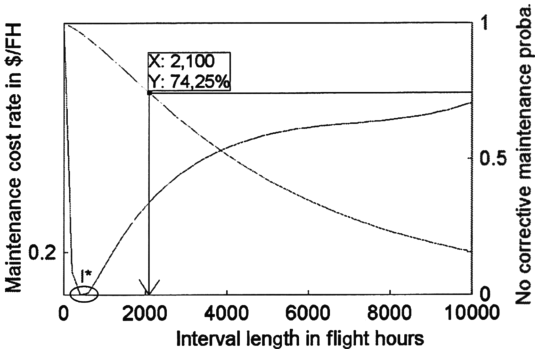

Fig. 5 Interval optimization as a function of failure-finding interval length.

also increases, which is logical. In fact, the main purpose of a failure-finding task is to prevent or at least reduce the risk of associated failure leading to corrective maintenance actions.

C. Optimization of Failure-Finding Frequency

If we suppose that the threshold δ is equal to 74.25%, then according to Fig. 5 the interval must be lower than 2100 FH. As a result, to minimize the cost rate the optimal interval I^* is equal to 500 FH. For this value the cost rate is $0.1872/\mathrm{FH}$.

VI. Conclusions

In this paper, we have introduced a new methodology for modeling hidden failure and process demand occurrences and the impacts on failure-finding and corrective maintenance. From this model, we provided cost and risk formulas and we derived an optimization problem to determine the optimal length of failure-finding task intervals. Moreover we have provided exact analytical formulas in the case of constant failure rate. We have illustrated these results with a typical failure-finding scenario on a pressure relief valve.

Future related work will attempt to extend our approach by relaxing one or more of the original assumptions. Our first attempt will be to address a broader problem that includes condition parameters directly linked to system degradation. Condition parameters could be any characteristic such as temperature, wear, humidity, sound, pressure, chemical concentration, or shock. As in a health management system, information provided by sensors on change in condition parameters can be used to optimize the next failure-finding task.

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